Sorting monoids on Coxeter groups

Florent Hivert[1] Anne Schilling[2] Nicolas M. Thiéry[2,3]

[1]LITIS/LIFAR, Université Rouen, France

[2]University of California at Davis, USA

[3]Laboratoire de Mathématiques d'Orsay, Université Paris Sud, France

FPSAC'10, San Francisco, August 3rd of 2010

arXiv:0711.1561 [math.RT] (FPSAC'06)
arXiv:0804.3781 [math.RT] (FPSAC'08)
arXiv:0912.2212 [math.CO] (FPSAC'10)
+ research in progress

Bubble (anti) sort algorithm

1234

Bubble (anti) sort algorithm

1234

Bubble (anti) sort algorithm

1243

Bubble (anti) sort algorithm

1423

Bubble (anti) sort algorithm

4123

Bubble (anti) sort algorithm

4123

Bubble (anti) sort algorithm

4132

Bubble (anti) sort algorithm

4312

Bubble (anti) sort algorithm

4312

Bubble (anti) sort algorithm

4321

Bubble (anti) sort algorithm

4321

Bubble (anti) sort algorithm

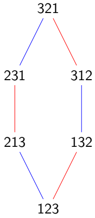
4321

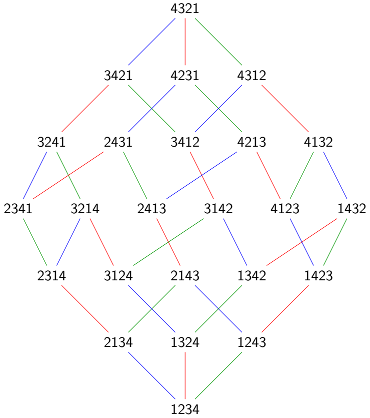
Underlying combinatorics: right permutahedron

Bubble (anti) sort algorithm

4321

Underlying combinatorics: right permutahedron

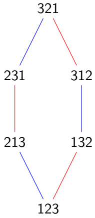


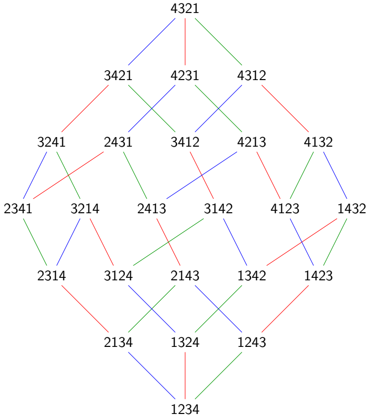


Bubble (anti) sort algorithm

4321

Underlying combinatorics: right permutahedron



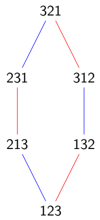


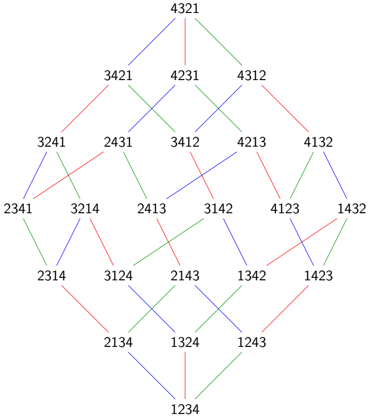
Elementary transpositions: $s_1, s_2, s_3, \ldots$

Bubble (anti) sort algorithm

4321

Underlying combinatorics: right permutahedron





Elementary transpositions: $s_1, s_2, s_3, \ldots$

Relations: $s_i^2 = 1, (s_1 s_2)^3 = 1, (s_2 s_3)^3 = 1, (s_1 s_3)^2 = 1$

Coxeter groups

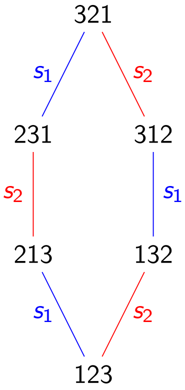
Definition (Coxeter group W)

Generators : $s_1, s_2, \ldots$ (simple reflections)

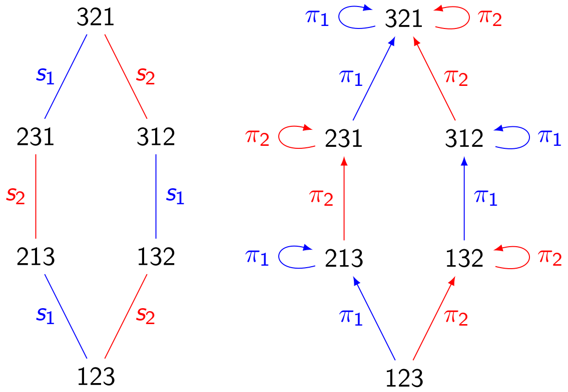
Relations: $s_i^2 = 1$ and $\underbrace{s_i s_j \cdots}_{m_{i,j}} = \underbrace{s_j s_i \cdots}_{m_{i,j}}$, for $i \neq j$

Reduced words

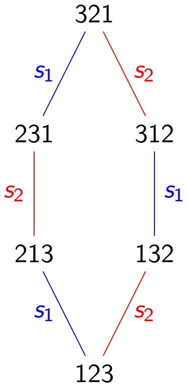
0-Hecke monoid

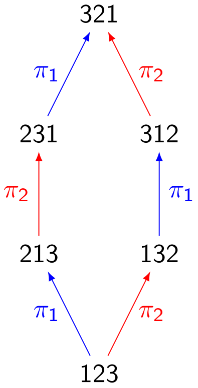


0-Hecke monoid

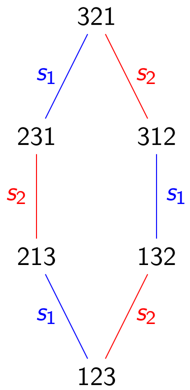


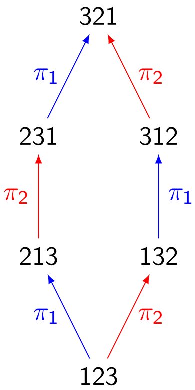
0-Hecke monoid

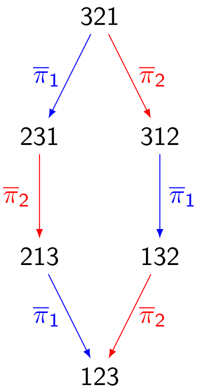




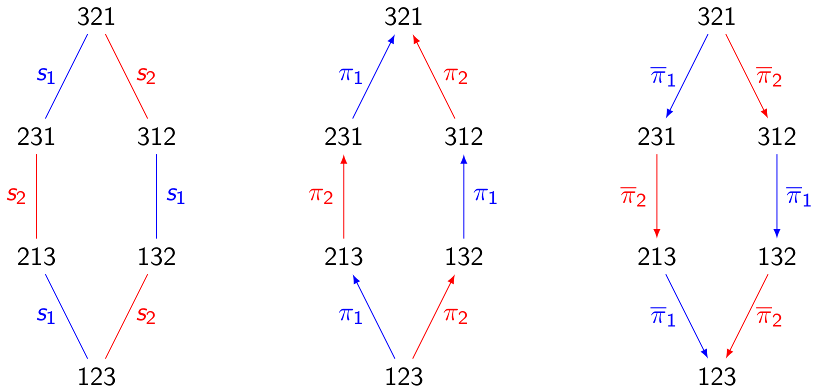
0-Hecke monoid







0-Hecke monoid



$\mathfrak{S}_3$ $H_0(\mathfrak{S}_3)$ $\overline{H}_0(\mathfrak{S}_3)$

0-Hecke monoid

Definition (0-Hecke monoid $H_0(W)$ of a Coxeter group W)

Generators : $\langle \pi_1, \pi_2, \dots \rangle$ (simple reflections)
Relations: $\pi_i^2 = \pi_i$ and braid relations

Theorem

$|H_0(W)| = |W|$
+ lots of nice properties

Motivation: simple combinatorial model (bubble sort)
appears in Iwahori-Hecke algebras, Schur symmetric functions,
Schubert, Macdonald, Kazhdan-Lusztig polynomials,
(affine) Stanley symmetric functions, mathematical physics,
Schur-Weyl duality for quantum groups, representations of $GL(\mathbb{F}_q)$,
...

0-Hecke monoid

Definition (0-Hecke monoid $H_0(W)$ of a Coxeter group W)

Generators : $\langle \pi_1, \pi_2, \ldots \rangle$ (simple reflections)
Relations: $\pi_i^2 = \pi_i$ and braid relations

Theorem

$|H_0(W)| = |W|$
+ lots of nice properties

Motivation: simple combinatorial model (bubble sort)
appears in Iwahori-Hecke algebras, Schur symmetric functions, Schubert, Macdonald, Kazhdan-Lusztig polynomials, (affine) Stanley symmetric functions, mathematical physics, Schur-Weyl duality for quantum groups, representations of $GL(\mathbb{F}_q)$, ...

0-Hecke monoid

Definition (0-Hecke monoid $H_0(W)$ of a Coxeter group W)

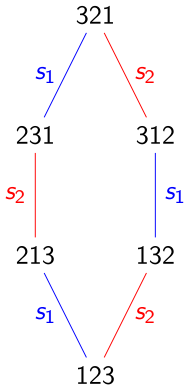
Generators : $\langle \pi_1, \pi_2, \ldots \rangle$ (simple reflections)
Relations: $\pi_i^2 = \pi_i$ and braid relations

Theorem

$|H_0(W)| = |W|$
+ lots of nice properties

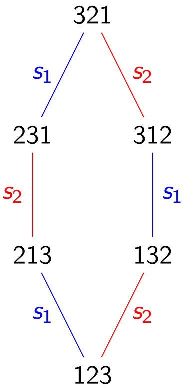
Motivation: simple combinatorial model (bubble sort)
appears in Iwahori-Hecke algebras, Schur symmetric functions, Schubert, Macdonald, Kazhdan-Lusztig polynomials, (affine) Stanley symmetric functions, mathematical physics, Schur-Weyl duality for quantum groups, representations of $GL(\mathbb{F}_q)$, ...

Classical orders on Coxeter groups



Right order

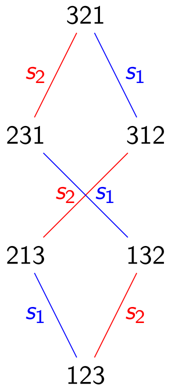
Classical orders on Coxeter groups



Right order

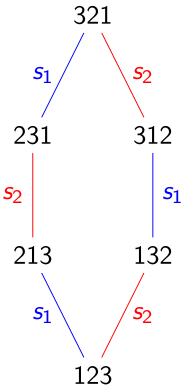
Prefix

Classical orders on Coxeter groups



Left order

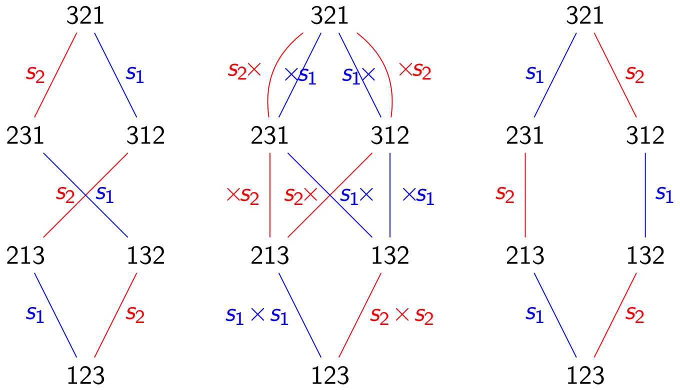
Suffix



Right order

Prefix

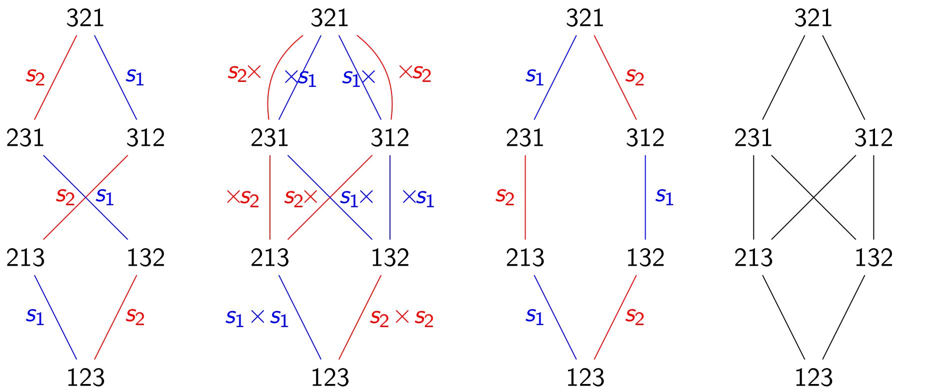
Classical orders on Coxeter groups



Left order | Left-Right order | Right order

Suffix | Factor | Prefix

Classical orders on Coxeter groups



Left order	Left-Right order	Right order	Bruhat order
Suffix	Factor	Prefix	Subword

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$

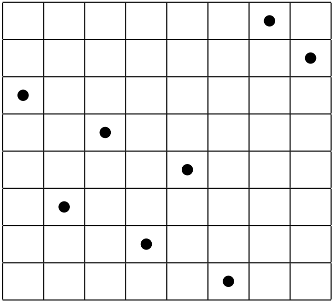
- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- {blocks of w}: sub-lattice of the Boolean lattice

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$



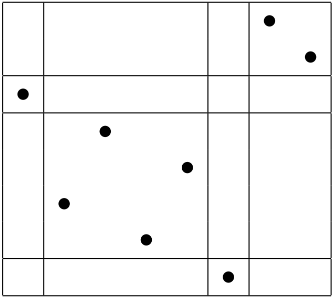
- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- $\{\texttt{blocks of } w\}$: sub-lattice of the Boolean lattice

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$



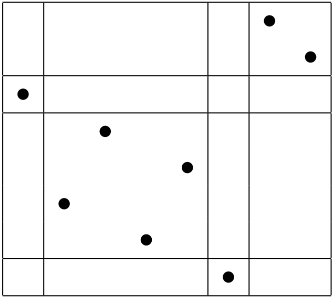
- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- {`blocks of` w}: sub-lattice of the Boolean lattice

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$



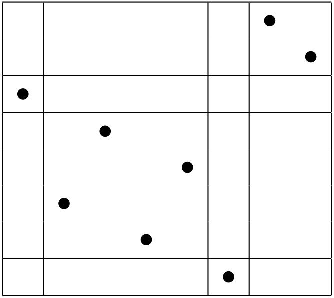
- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- $\{\texttt{blocks of } w\}$: sub-lattice of the Boolean lattice

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$



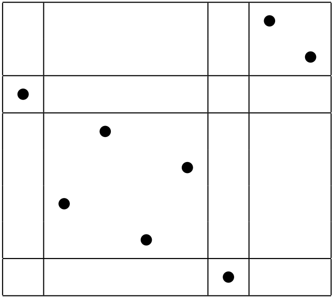
- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- {blocks of w}: sub-lattice of the Boolean lattice

Blocks of permutations

Definition (Block of a permutation w)

- Type A: sub-permutation matrix
- Type free: J, K such that $W_J w = w W_K$

- Example: $w := 36475812$



- Simple permutation: cf. [Albert, Atkinson 05] + dim 2 posets
- {`blocks of` w}: sub-lattice of the Boolean lattice

The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block

Theorem

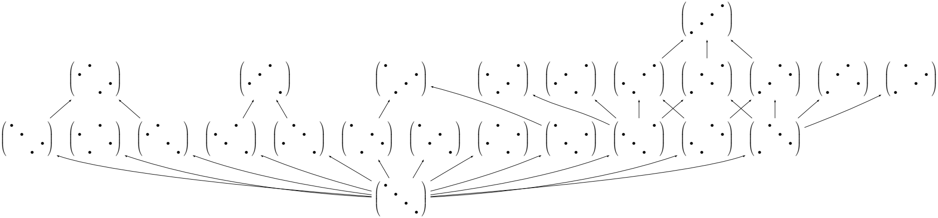
- *Intervals are lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block



Theorem

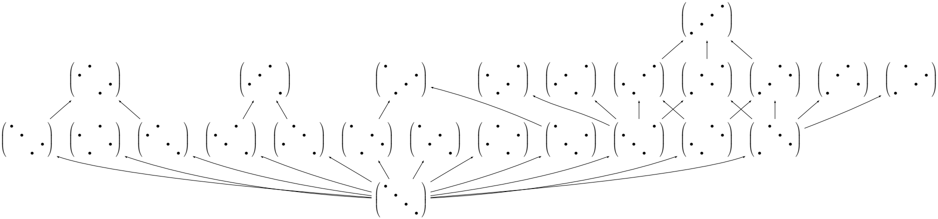
- *Intervals are lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block



Theorem

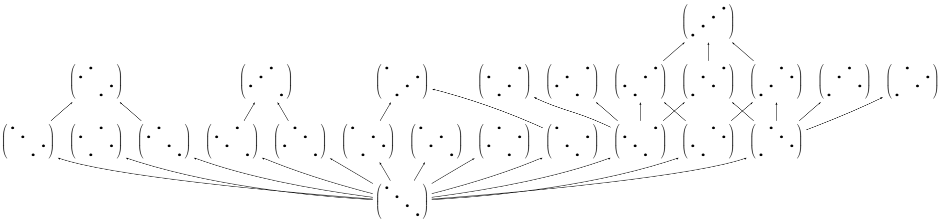
- *Intervals are lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block



Theorem

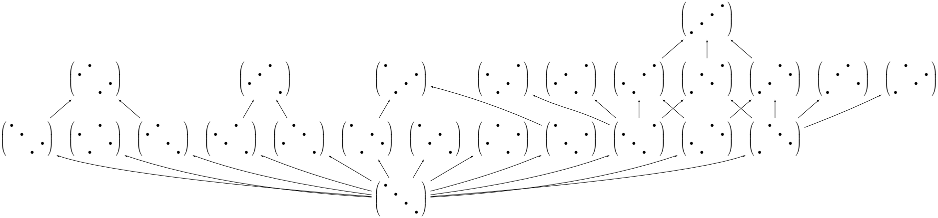
- *Intervals are lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block



Theorem

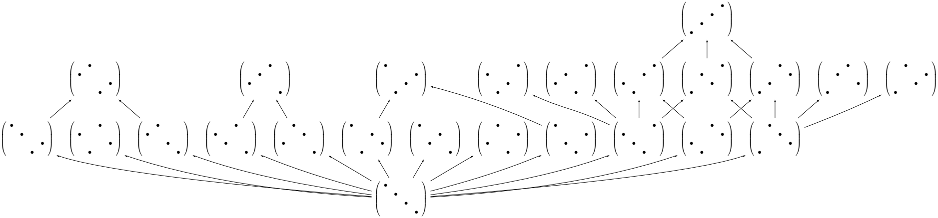
- *Intervals are (distributive?) lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

The cutting poset

Definition (HST09: Cutting poset $(W, \sqsubseteq)$)

$u \sqsubseteq w$ if $u = w^J$ with J block

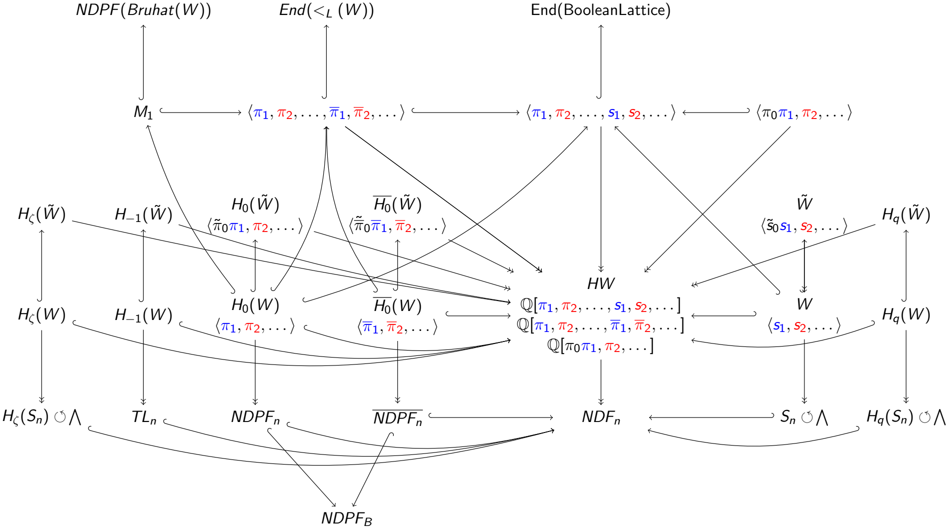


Theorem

- *Intervals are (distributive?) lattices*
- *Möbius function: inclusion-exclusion along minimal blocks*

- Meet-semi lattice?

The Big Picture



The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$

$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$

$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$
$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ *admits* $|W|$ *simple / indecomposable projective modules*

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$
$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$
$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$
$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

The biHecke monoid

Question

Size of $M(W) = \langle \pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots \rangle$
$|M(S_n)| = 1, 3, 23, 477, 31103, ?$

- How to attack such a problem?
- Generators and relations?
- Representation theory?

Theorem (HST08)

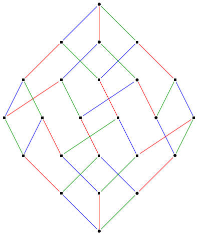
$M(W)$ admits $|W|$ simple / indecomposable projective modules

- Why do we care?

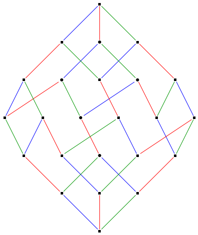
$$|M(W)| = \sum_{w \in W} \dim S_w . \dim P_w$$

Key combinatorial lemma

$1 3 \overline{2}$



$\longmapsto$



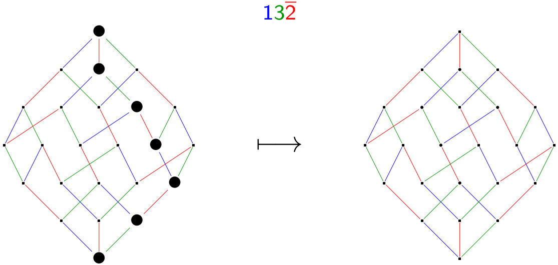
Lemma

For $f \in M(W)$ and $w \in W$: $\quad (s_i w).f = w.f \quad \text{or} \quad s_i\,(w.f)$

Proof.

Exchange property / associativity □

Key combinatorial lemma



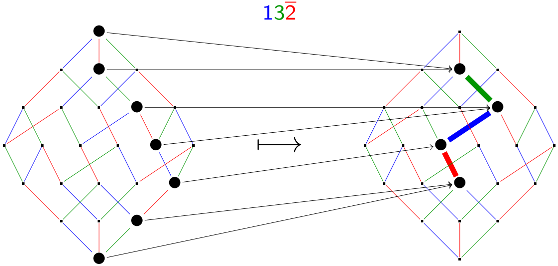
Lemma

For $f \in M(W)$ and $w \in W$: $\quad (s_i w).f = w.f \quad$ *or* $\quad s_i\,(w.f)$

Proof.

Exchange property / associativity □

Key combinatorial lemma



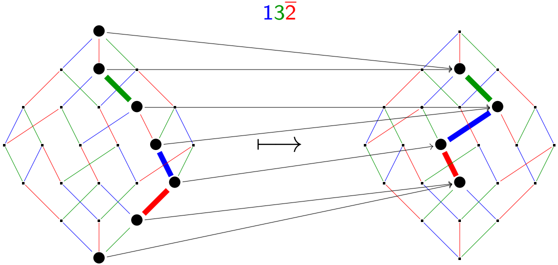
Lemma

For $f \in M(W)$ and $w \in W$: $\quad (s_i w).f = w.f \quad$ *or* $\quad s_i\,(w.f)$

Proof.

Exchange property / associativity □

Key combinatorial lemma



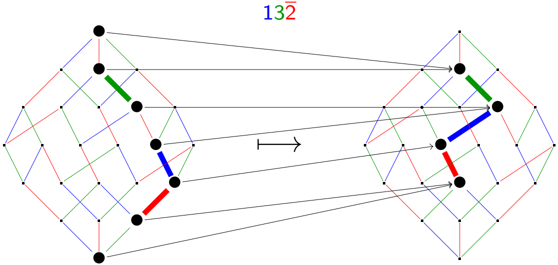
Lemma

For $f \in M(W)$ and $w \in W$: $(s_i w).f = w.f$ or $s_i\,(w.f)$

Proof.

Exchange property / associativity □

Key combinatorial lemma



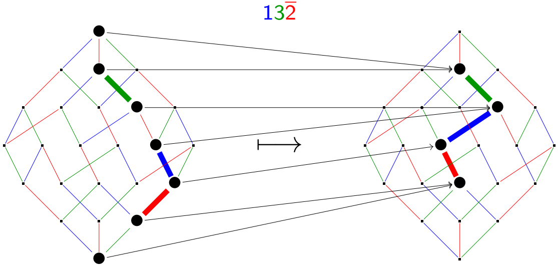
Lemma

For $f \in M(W)$ and $w \in W$: $\quad (s_i w).f = w.f \quad$ *or* $\quad s_i\,(w.f)$

Proof.

Exchange property / associativity □

Key combinatorial lemma



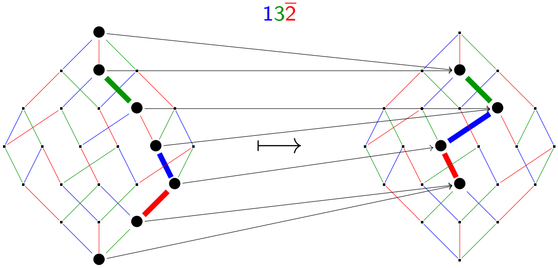
Lemma

For $f \in M(W)$ and $w \in W$: $\quad (s_i w).f = w.f \quad$ *or* $\quad s_i\,(w.f)$

Proof.

Exchange property / associativity □

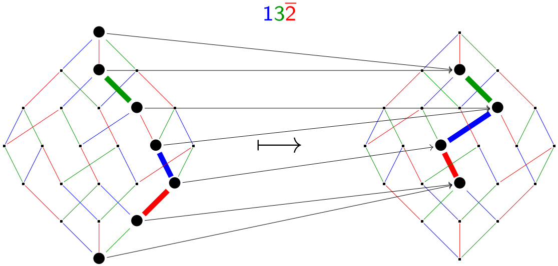
Key combinatorial lemma



Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

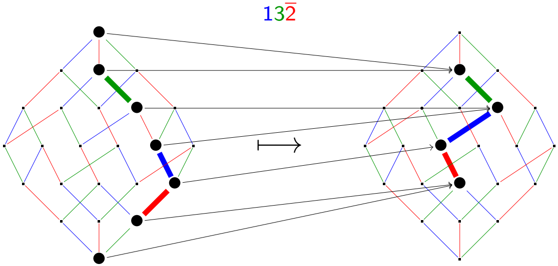
Key combinatorial lemma



Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

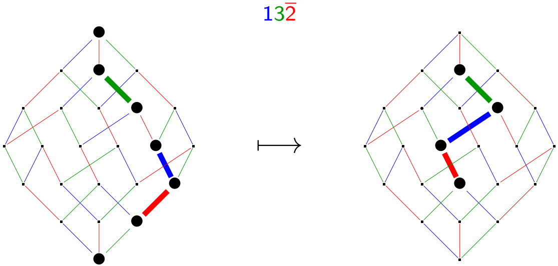
Key combinatorial lemma



Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

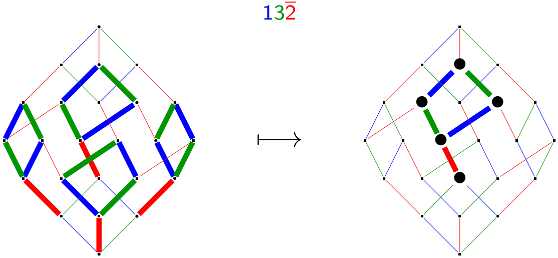
Key combinatorial lemma



Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

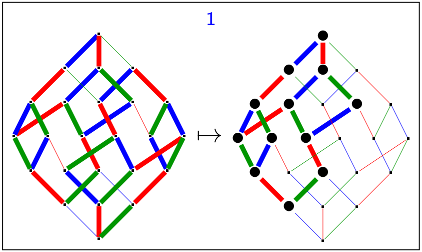
Key combinatorial lemma



Corollary

- *Preservation of left order:* $u \leq_L v \Longrightarrow u.f \leq_L v.f$
- *Preservation of Bruhat order:* $u \leq_B v \Longrightarrow u.f \leq_B v.f$
- $M(W)$ *is aperiodic*
- f *in* $M(W)$ *is determined by its fibers and* $f(1)$

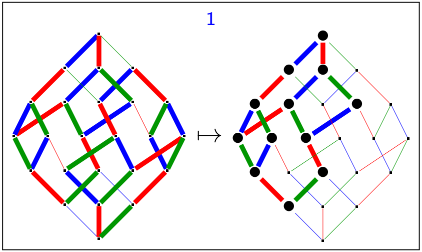
Some elements of the monoid

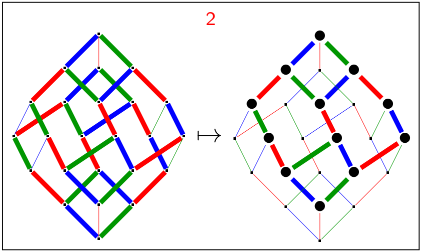


Lemma

The image set of an idempotent is an interval in left order

Some elements of the monoid

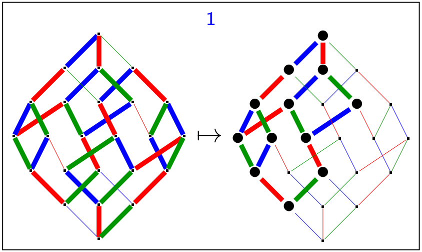


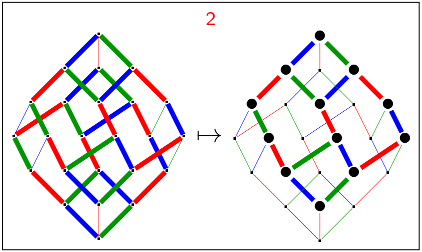


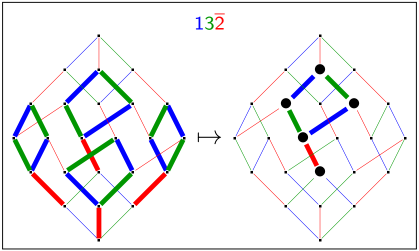
Lemma

The image set of an idempotent is an interval in left order

Some elements of the monoid



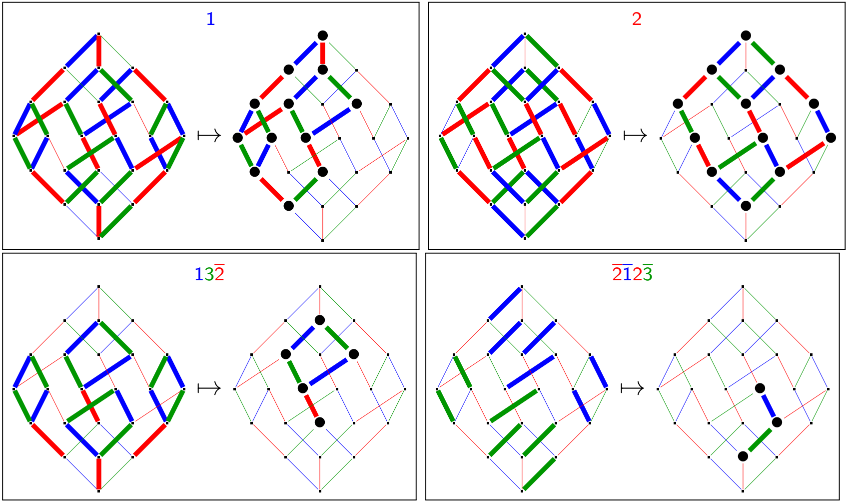




Lemma

The image set of an idempotent is an interval in left order

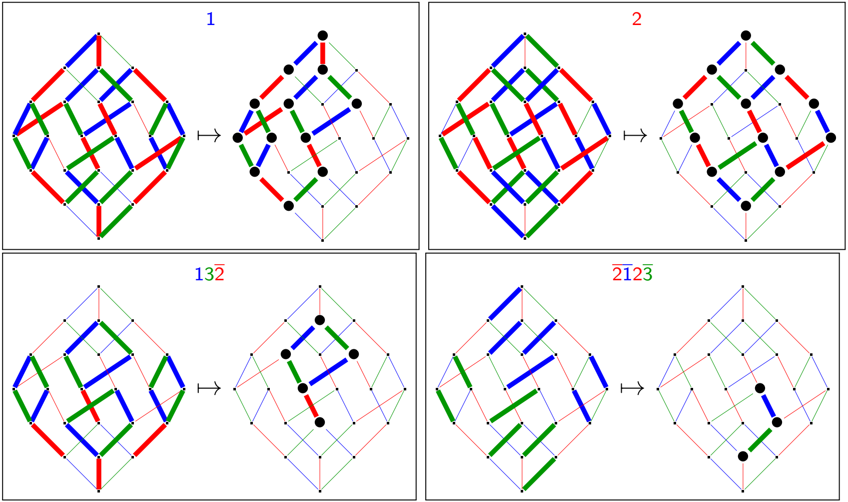
Some elements of the monoid



Lemma

The image set of an idempotent is an interval in left order

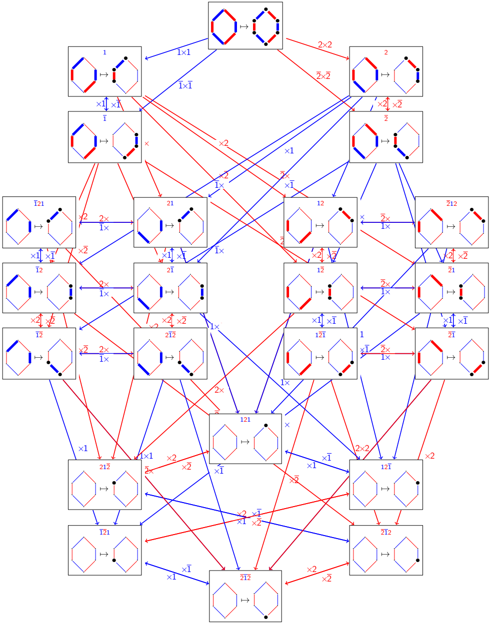
Some elements of the monoid



Lemma

The image set of an idempotent is an interval in left order

Green relations



Green relations

Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Green relations

Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Green relations

Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Green relations

Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Green relations

Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Green relations

Theorem

- *Regular $\mathcal{J}$-classes are indexed by W*
- *$\mathcal{J}$-order on regular classes: left-right order on W*
- *$\mathcal{R}$-classes: intervals in right order on W*
- *$\mathcal{R}$-order on regular $\mathcal{R}$-classes: $\approx$ right order on W*
- *$\mathcal{L}$-order on regular $\mathcal{L}$-classes: $\approx$ left order on W*

Problems

- *$\mathcal{L}$,$\mathcal{R}$,$\mathcal{J}$-order between non regular classes?*
- *$\mathcal{L}$-classes?*

Representation theory of $M(W)$

Corollary

$M(W)$ admits $|W|$ simple modules / indecomposable projective modules

Problem

Dimension of simple and indecomposable projective modules?

Representation theory of $M(W)$

Corollary

$M(W)$ admits $|W|$ simple modules / indecomposable projective modules

Problem

Dimension of simple and indecomposable projective modules?

The "Borel" submonoid M_1

Definition

Submonoid $M_1 := \{f \in M, f(1) = 1\}$

Properties

- *Weakly increasing and contracting on Bruhat* $\Longrightarrow$ $\mathcal{J}$*-trivial*
- *Idempotents:* $(e_w)_{w \in W}$
- *Generated by* e_w *for* w *grassmanian, e.g. atom for* $(W, \vee_L)$
- $|W|$ *simple modules of dimension* 1
- *Semi-simple quotient: monoid algebra of* $(W, \vee_L)$
- *Conjugacy order among idempotents:* $<_L$
- $\dim P_w = |\{f \in M_1,\ f(w) \leq_L w\}| = ?$

Problem

Inducing these results to M*?*

The “Borel” submonoid M_1

Definition

Submonoid $M_1 := \{f \in M, f(1) = 1\}$

Properties

- *Weakly increasing and contracting on Bruhat* $\Longrightarrow$ $\mathcal{J}$*-trivial*
- *Idempotents:* $(e_w)_{w \in W}$
- *Generated by* e_w *for* w *grassmanian, e.g. atom for* $(W, \vee_L)$
- $|W|$ *simple modules of dimension* 1
- *Semi-simple quotient: monoid algebra of* $(W, \vee_L)$
- *Conjugacy order among idempotents:* $<_L$
- $\dim P_w = |\{f \in M_1,\ f(w) \leq_L w\}| =?$

Problem

Inducing these results to M?

The "Borel" submonoid M_1

Definition

Submonoid $M_1 := \{f \in M, f(1) = 1\}$

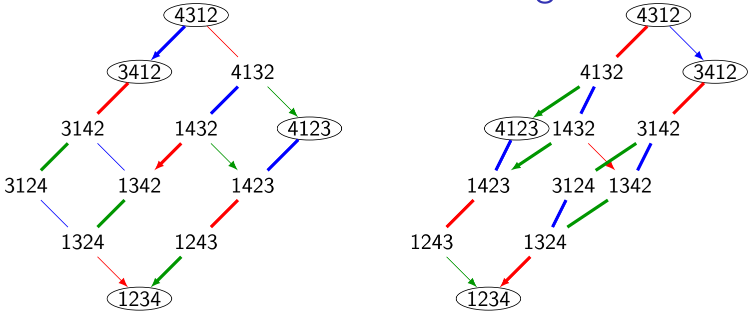
Properties

- *Weakly increasing and contracting on Bruhat* $\Longrightarrow$ $\mathcal{J}$*-trivial*
- *Idempotents:* $(e_w)_{w \in W}$
- *Generated by* e_w *for* w *grassmanian, e.g. atom for* $(W, \vee_L)$
- $|W|$ *simple modules of dimension* 1
- *Semi-simple quotient: monoid algebra of* $(W, \vee_L)$
- *Conjugacy order among idempotents:* $<_L$
- $\dim P_w = |\{f \in M_1,\ f(w) \leq_L w\}| =?$

Problem

Inducing these results to M?

$\mathcal{R}$-classes and translation algebras

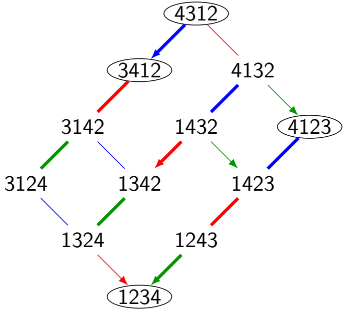


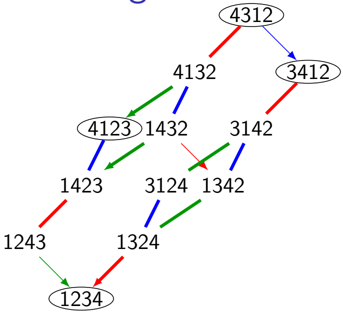
Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\}$ $\Longrightarrow$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all P_J $\Longrightarrow$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

$\mathcal{R}$-classes and translation algebras



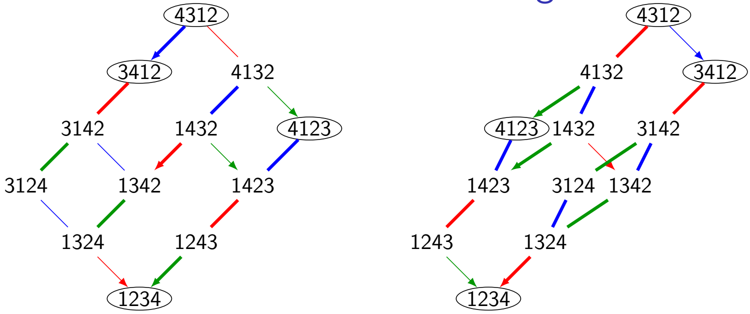


Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \dots, \overline{\pi}_1, \overline{\pi}_2, \dots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\}$ $\implies$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all P_J $\implies$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

$\mathcal{R}$-classes and translation algebras

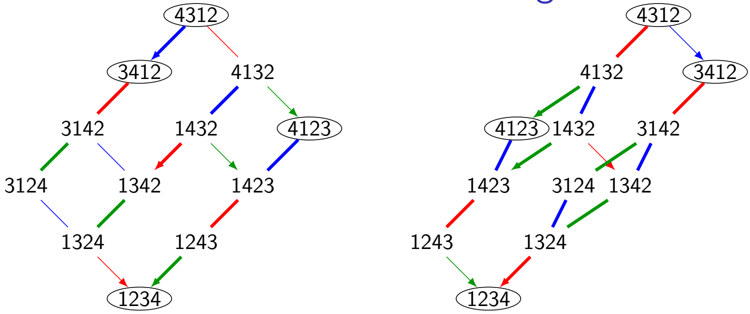


Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \dots, \overline{\pi}_1, \overline{\pi}_2, \dots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\} \implies$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all $P_J \implies$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

$\mathcal{R}$-classes and translation algebras

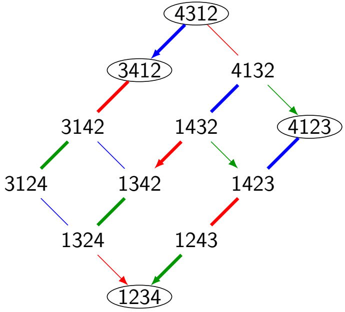


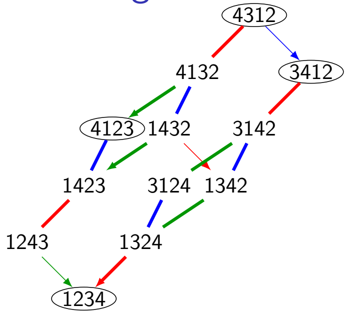
Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \dots, \overline{\pi}_1, \overline{\pi}_2, \dots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\}$ $\Longrightarrow$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all P_J $\Longrightarrow$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

$\mathcal{R}$-classes and translation algebras



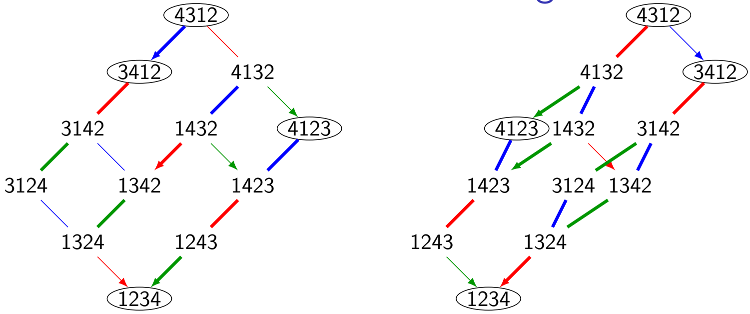


Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\}$ $\implies$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all P_J $\implies$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

$\mathcal{R}$-classes and translation algebras

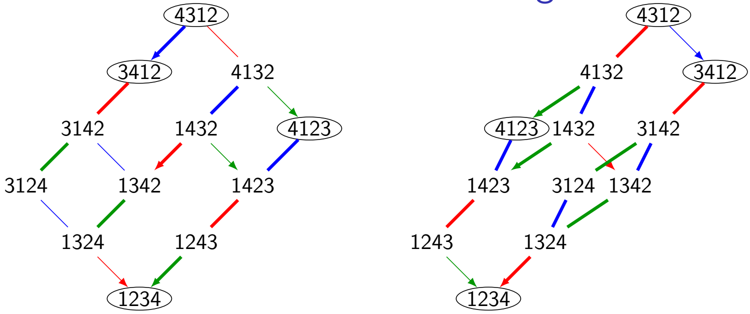


Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \dots, \overline{\pi}_1, \overline{\pi}_2, \dots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\}$ $\implies$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all P_J $\implies$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

$\mathcal{R}$-classes and translation algebras



Definition (Translation algebra)

$\mathcal{H}W^{(w)} := \mathbb{Q}[\pi_1, \pi_2, \ldots, \overline{\pi}_1, \overline{\pi}_2, \ldots]$ acting on $\mathbb{Q}.[1, w]_R$

- Blocks: $J = \{\}, \{1, 2\}, \{3\}, \{1, 2, 3\} \implies$ Submodules P_J
- $\mathcal{H}W^{(w)}$: max. algebra stabilizing all $P_J \implies$ Repr. theory
- $\mathcal{H}W^{(w)}$ quotient of $\mathbb{Q}[M(W)]$; top: simple module S_w of M
- Dimension: inclusion-exclusion along the cutting poset
- Generating series calculation?

Work in progress

- $\mathcal{L}$-classes? Projective modules? Cartan Matrix?
- Generalization to $\mathcal{R}$-trivial and aperiodic monoids
  (collaboration with Denton and Berg, Bergeron, Saliola)
- Fast implementation in Sage
  (interface with `Semigroupe`, ...)

Sage-Combinat meeting tonight

Sage's mission:

"To create a viable high-quality and open-source alternative to Maple™, Mathematica™, Magma™, and MATLAB™"

...

"and to foster a friendly community of users and developers"

Tonight, Thorton Hall, Room 326

- 7pm-8pm: Introduction to `Sage` and `Sage-Combinat`
- 8pm-10pm: Help on installation & getting started
  Bring your laptop!
- Design discussions